PERGAMON

International Journal of Solids and Structures 37 (2000) 991–1001

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Study of the fracture toughness by finite element methods

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Received 12 December 1997; in revised form 29 January 1999

Abstract

In this work, the critical stress intensity factor of a centre cracked plate is analysed by finite element methods to study its dependence upon the geometry of the plate and to search under what conditions it can be identified with the fracture toughness of the material. This will be useful for a better understanding of the theoretical concept of fracture toughness and help to determine its magnitude both by finite element procedures and experimentally.

Keywords: Toughness; Finite elements

1. Introduction

When crack propagation takes place in a structural element it is because some stress at the crack tip, or in the vicinity of a defect of the crystalline lattice, exceeds the tensile strength of the material.

Consequently, the main question should be to analyse the stress distribution within the region where a crack is possible to appear and around any crack-like defect. Nevertheless, this information is usually only the first step for the designer for, eventually, he is interested in the loads that the structure can withstand. Therefore, as the loads are applied in general away from the failure regions, it is of paramount importance to understand the relation between the applied loads and the stress distribution in the region of a crack or around any defect of the material. Thus, the resultant stresses can be studied as a function of the magnitude of the applied loads and crack expansion can be avoided from the design.

In this context, the strength of the material is the ultimate stress beyond which any stress at the crack tip originates that the crack progress, whereas the resistance that the material opposes against the crack propagation, in terms of the applied load remote from the crack, is represented by the so-called fracture toughness, as defined by fracture mechanics.

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0020-7683/00/$ - see front matter
PII: S0020-7683(99)00108-0

This makes clear the significance of the fracture toughness concept and reveals how crucial it is in the fracture mechanics field. Hence, when dealing with any aspect of fracture, it is vital to understand it precisely and, moreover, to quantify its actual magnitude for a particular material.

As a result, measuring the fracture toughness of a material became an engineering necessity from the very beginning of the fracture mechanics development. Indeed there is an abundant and still increasing literature on these topics (see: Sinclair et al., 1996; Dao, 1996; Sheinman and Kardomateas, 1997; Chen and Hsu, 1997; Liu et al., 1996; Kim, 1996, among many others). Besides, fracture toughness is a central subject within all the texts devoted to fracture, like those by Knott (1973), Parker (1981), Atkins and Mai (1985), Kanninen and Popelar (1985) and Broek (1988). Furthermore, many ways have been described to determine fracture toughness (Li, 1996;, Oh, 1996; Seibi and Al-Alawi, 1996; Li and Bakker, 1997), some of which were adopted as standard procedures.

The purpose of this work is to verify by means of the finite element analysis, the fracture toughness concept and contribute to its full understanding. Also, it is hoped that applying finite element methods will help to determine the fracture toughness itself as well as to design the test specimens which may lead to find it out in the laboratory.

2. The fracture toughness

When a structure is subjected to external loading, it is obvious that the stress distribution in any structural element depends on the loading, on the material characteristics and on the geometry of the element.

Then, if a crack appears somewhere within the element, it provokes a stress redistribution, especially at the crack region. In particular, it is proved that the stress at any point close to the crack tip, where the crack can progress, is proportional to a factor, usually called ‘the stress intensity factor’. For a centre-cracked infinite sheet, if it is thick enough so that the plate behaves as plain strain, the stress intensity factor is

$$K = \sigma_{\mathrm{a}}\sqrt{\pi a}, \tag{1}$$

where σ_{a} is the applied tension stress remote from the crack and a is the semi-crack length.

Accordingly, the longitudinal tensile stress at the crack tip, where the crack can progress, is

$$\sigma = \sigma_{\mathrm{a}}\sqrt{\pi a}F, \tag{2}$$

F being a function of the position of the point at which the stress is referred.

If the applied tension is increased, the stress at the crack tip grows and will eventually reach the ultimate tensile stress of the material, with which the crack expands. Then the applied stress becomes critical

$$\sigma = \sigma_{\mathrm{u}} = \sigma_{\mathrm{ac}}\sqrt{\pi a}F \tag{3}$$

and the stress intensity factor acquires a critical value

$$K_{\mathrm{c}} = \sigma_{\mathrm{ac}}\sqrt{\pi a}, \tag{4}$$

in which σ_{ac} is the critical applied stress for a crack of length $2a$.

Fracture mechanics theory proves the important fact that this critical stress intensity factor does not depend on the crack size but, if the sheet is thick enough, remains constant while the crack varies. An evident consequence from Eq. (4) is that the critical applied stress diminishes if the crack grows.

Effectively, Eq. (4) shows the diminution of the applied critical stress in relation to the increase of the crack length. In other words, this is the quantification of the obvious fact that, once a crack starts progressing, some unloading is necessary before it stops.

If the structural element is not an infinite sheet, the value of K from Eq. (1) must be multiplied by a geometry correction factor Y which, in general, depends on the crack size and on the geometry of the element. In this case, the stress at the crack tip would be given by

$$\sigma' = Y\sigma_{\mathrm{a}}'\sqrt{\pi a}F, \tag{5}$$

σ_{a}' being the applied tension stress over the element whose geometry factor is Y. Thus, when the ultimate strength of the material is reached,

$$\sigma' = \sigma_{\mathrm{u}} = Y\sigma_{\mathrm{ac}}'\sqrt{\pi a}F \tag{6}$$

and the critical stress intensity factor is

$$K_{\mathrm{c}}' = Y\sigma_{\mathrm{ac}}'\sqrt{\pi a}, \tag{7}$$

where σ_{ac}' is the critical applied stress on the element with a crack of size $2a$.

Now comparing Eq. (3), referred to a thick infinite plate, and Eq. (6), referred to any structural element, allows us to deduce that the critical stress intensity factor is worth the same in all cases

$$K_{\mathrm{c}} = \sigma_{\mathrm{ac}}\sqrt{\pi a} = Y\sigma_{\mathrm{ac}}'\sqrt{\pi a}, \tag{8}$$

no matter what sort of structural element may be considered.

As a consequence of all this, the critical stress intensity factor of a centre-cracked infinite sheet can be thought of as a material fracture parameter, as long as it behaves as plain strain, and so it is called the 'plain strain fracture toughness of the material'. Hence, the importance of the plate thickness is derived, for it has to guarantee a plain strain behaviour so that the critical stress intensity factor can be effectively identified with the fracture toughness.

In what follows, the fracture toughness will be talked of whether as the critical stress intensity factor of a centre-cracked thick infinite sheet, as shown in Eq. (4), or as the critical stress intensity factor of a centre-cracked thick finite plate, as shown in Eq. (7). In the latter case, the geometry correction factor will be determined using the expression previously derived and confirmed by Feddersen (1966), Isida (1971) and Mínguez (1993):

$$Y = \sqrt{\sec\frac{\pi a}{2w}}, \tag{9}$$

where $2w$ is the finite plate width.

Accordingly, the dependence of the critical stress intensity factor upon the plate geometry becomes a question of major interest, to understand under what conditions it can be identified with the fracture toughness of the material.

3. Finite element models

The analysis of the fracture toughness carried out in this work was implemented over three plates modelled with finite elements. The three plates were modelled by means of 8 node three-dimensional elements (type C3D8) provided by the *ABAQUS* finite element computer package.

The first plate was a square of 600 cm side and varying thickness, while the other two were rectangles of heigth 300 cm and widths 100 cm and 160 cm, respectively. Every plate had a transverse central crack and, consequently, double axial symmetry allowed us to work with models that represented only one quarter of the actual plates. This was very advantageous, for it saved time and the plates could be divided for the finite element analysis into an appropriate mesh. Effectively, the smaller and more numerous the elements of the mesh are made, the more accurate are the results obtained, but the computation time increases. However, as stress distributions become less crucial as distance from the crack boundary increases, it is not necessary to extend the fine mesh all over the plate. It is best to use a fine mesh with numerous small elements near the crack boundary and fewer and larger elements away from it. This was achieved by using ever longer elements, as shown in Figs 1 and 2, where it can also be seen that the aspect ratio never exceeded five to one, which is a requirement of the *ABAQUS* finite element package.

Besides, an essential consideration was that if the results for the stress distributions in the vicinity of the crack in the three plates were to be consistent, the element size of the mesh in those regions needed to be always the same. Therefore, the elements in the vicinity of the crack were squares of 2 cm in all three cases.

The left vertical edge of the sheet is a symmetry axis and so it had to be kept straight during the loading. For this reason, the nodes along its line were anchored so that they could not move horizontally. Similarly, the nodes of the bottom horizontal edge were anchored so that they could not move vertically. Crack growth was simulated by consecutively releasing the left-hand points of the bottom edge along the desired half crack length a, to allow their free movement, as occurs in real conditions.

These models accurately reflected the geometry of the plates with their cracks, upon which a tensile stress $\sigma_a = 10000$ N/cm^2 was applied uniformly distributed over the top edge.

In all cases, the plate thickness was varied and the effect of this variation carefully analysed in order

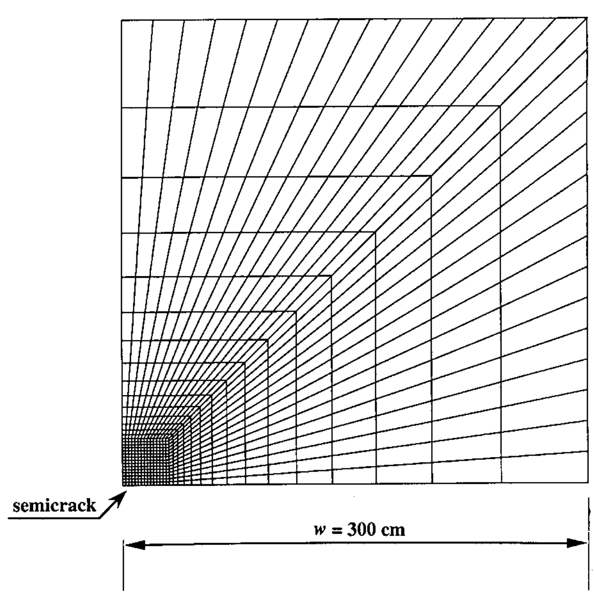

Fig. 1. Finite element model representing one quarter of the largest plate. The elements at the crack region are 2 cm squares.

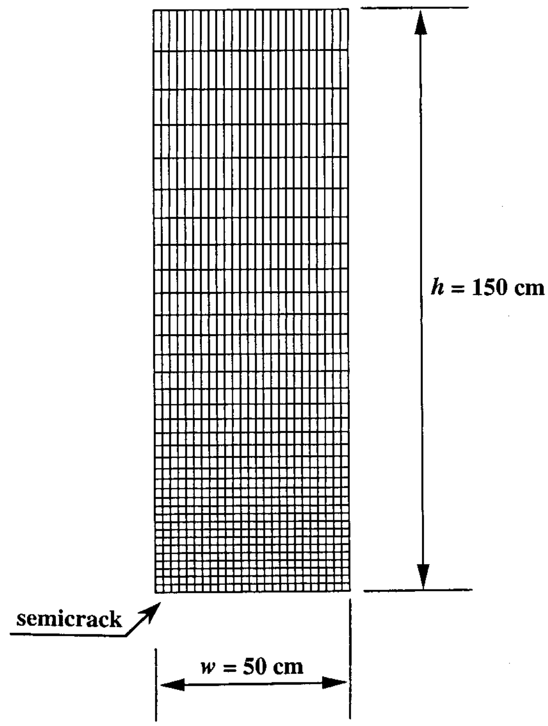

Fig. 2. Finite element model representing one quarter of a narrow plate. The elements at the crack region are 2 cm squares.

to study its influence over the critical stress intensity factor and to determine whether plain strain conditions were applied or not.

As for the material of the plates, they were considered to be made from a perfectly elastic alloy with a modulus of elasticity $E = 7.10^6$ N/cm^2 and a Poisson's ratio $\nu = 0.33$. These figures correspond to a real aluminium alloy, L165/Cu 4%. The ultimate tensile strength was $\sigma_u = 44500$ N/cm^2.

4. The infinite plate

The first plate was the largest one and was intended to help the understanding of the fracture toughness concept as a property of the infinite plate. It was subjected to longitudinal tension, for which the plate thickness was varied from $t = 1$ cm to $t = 50$ cm and different crack lengths were considered between $2a = 4$ cm and $2a = 56$ cm at the center of the total plate width $2w = 600$ cm.

Then, from the stress at the crack tip under the applied tension, the critical stress intensity factor was found out according to Eqs. (4) and (7), after calculating the applied tension needed for the stress at the crack tip to reach the ultimate tensile strength of the material.

In Fig. 3, the critical stress intensity factor of the plate with a crack of length $2a = 56$ cm, is shown versus the plate thickness, in the two assumptions, first, considering the actual dimensions of the plate ($2w = 600$ cm) and applying Eqs. (7) and (9) and, second, considering the plate as infinite, this is taking the geometry correction factor as $Y = 1$, with which Eq. (7) transforms into Eq. (4). The validity of this approach will be discussed later. This figure also demonstrates that the critical stress intensity factor of

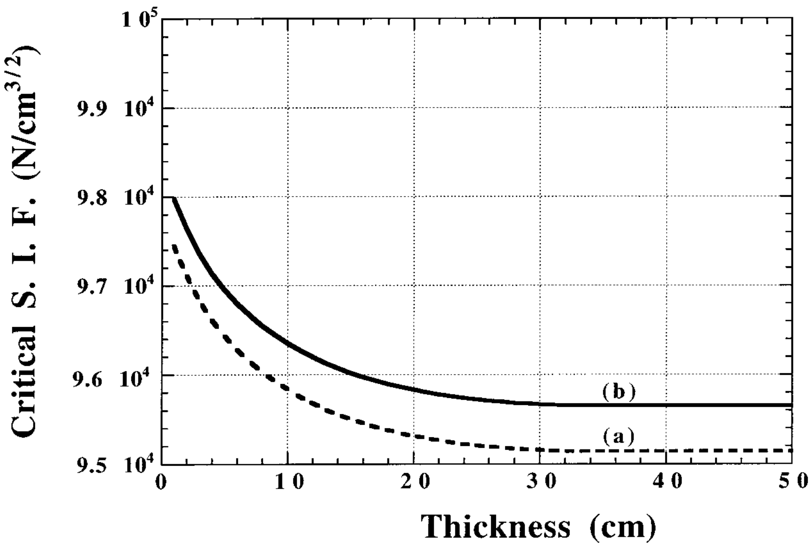

Fig. 3. Critical stress intensity factor as deduced from the largest sheet (600×600 cm^2) with a crack of size $2a = 56$ cm, versus the thickness: (a) as if the plate were infinite, and (b) considering the actual width of the plate.

the plate depends on the plate thickness and becomes constant only after a certain thickness, when the plate behaves as plain strain.

Fig. 4 shows the critical stress intensity factor, as calculated from the same plate with two different crack lengths, and shows that the thickness, beyond which the plate performs as plain strain and the critical stress intensity factor becomes a material property, depends on the crack length.

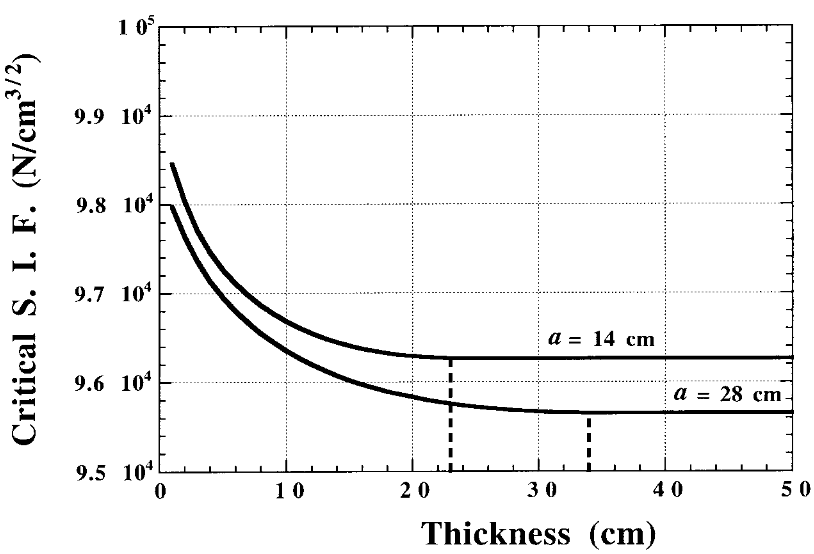

Fig. 4. Critical stress intensity factor as calculated from the largest sheet with two different cracks, versus the thickness.

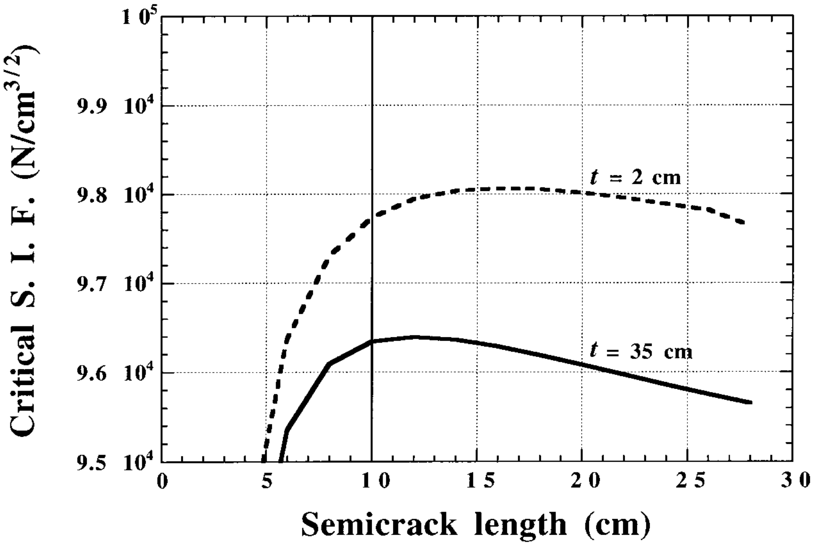

Fig. 5. Critical stress intensity factor as deduced from the largest sheet with two extreme thicknesses, against the crack size.

Finally, in Fig. 5, the critical stress intensity factor is plotted against the crack length for two extreme thicknesses, one long before plain strain conditions are reached and the other after that circumstance, when the critical stress intensity factor can be named as the fracture toughness of the material. This will allow us to know the required thickness of the plate when the fracture toughness is going to be determined in practice. From the finite element analysis, it is also deduced that over a crack size of, say $2a = 20$ cm, the stress intensity factor remains nearly constant, as derived from theory, which proves that it is a material parameter and can be properly called 'fracture toughness'.

5. Finite plates

Similar tests were carried out over two smaller plates to study under what conditions the fracture toughness can be determined on plates of limited dimensions, after accounting for the corresponding geometry correction factor, as determined in Eq. (9).

Fig. 6 shows that the critical stress intensity factor calculated from a narrow plate depends on the plate thickness up to a certain thickness, as happens when deducing it from an infinite sheet. Beyond that particular value, the critical stress intensity factor effectively remains nearly constant in this case also and is equal to the fracture toughness of the material.

In Fig. 7, the stress intensity factor is represented for two extreme thicknesses when the crack grows. Again, the stress intensity factor behaviour over a determined crack size approaches the invariability, which means it is a fracture parameter of the material, in accordance with the theory.

The following section is dedicated to a general discussion of all the results, including Fig. 8, in which the lowest thickness to guarantee a plain strain performance of two plates is represented as a function of the crack size.

Such a detailed discussion in quantitative terms will allow us to draw some interesting and useful conclusions.

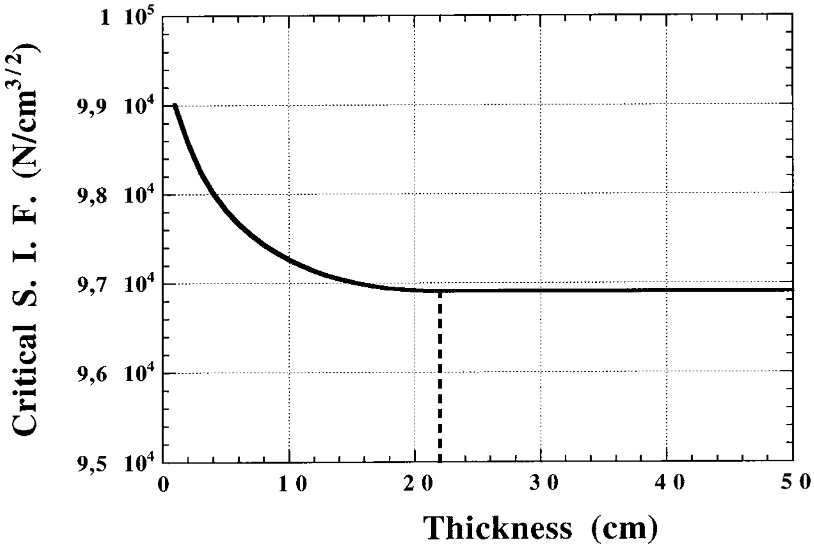

Fig. 6. Critical stress intensity factor as deduced from the narrow sheet ($100 \times 300\ cm^2$) with a crack of size $2a = 28$ cm, versus the thickness.

6. Discussion

The fracture toughness of the material was first determined by submitting to tension the largest plate with a central crack of 56 cm, which represented 9% of the full width of the plate. Fig. 3 shows that considering the plate as infinite and calculating straightforwardly ($Y = 1$), the critical stress intensity factor results in an underestimate of its magnitude by 0.5%, compared to the results obtained when

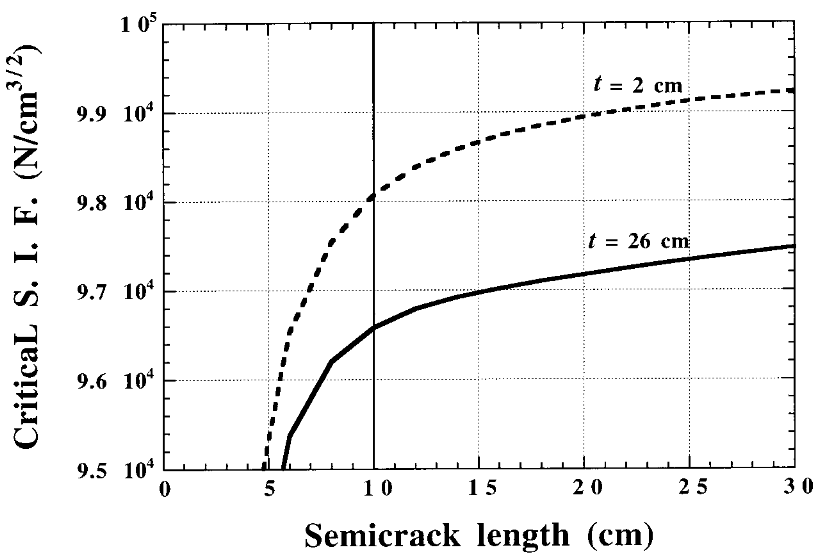

Fig. 7. Critical stress intensity factor as calculated from the narrow sheet with two extreme thicknesses, against the crack size.

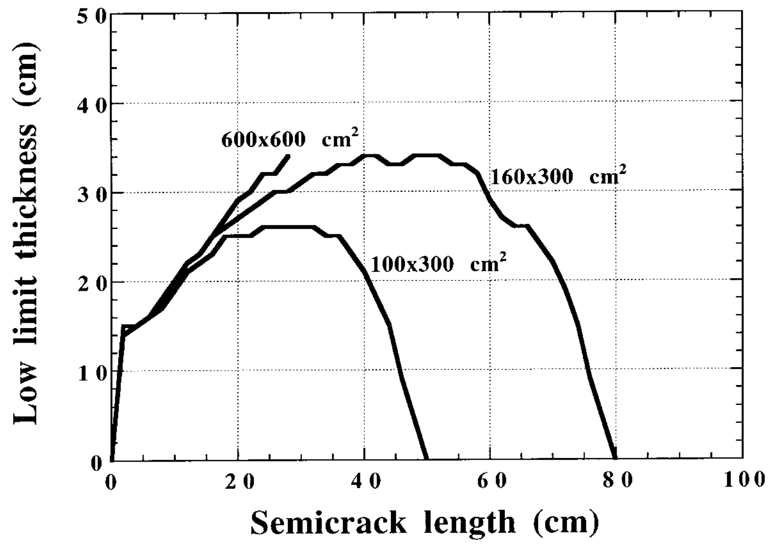

Fig. 8. Lowest thickness of two plates needed for their plain strain perfomance at the crack region, as a function of the crack size.

accounting for the actual value of the geometry correction factor of the finite plate. This underestimation is kept unaltered, whatever the thickness of the plate may be, and so the fracture toughness suffers the same underestimation due to our considering the finite plate as an infinite one.

In particular, it is worthwhile to realise that if the disparity between the actual fracture toughness and its estimate has to be kept under 5%, the ratio of the crack size to the width of the finite plate cannot exceed 0.7, as deduced from Eq. (9), which yields the geometry correction factor. This is an extremely high value for the allowed a/w ratio, which leads to the conclusion that substituting the fracture toughness of the infinite sheet with a finite one in the definition is quite acceptable for, in practice, the cracks are always well under 70% of the plate width. In theory, this is a very important feature of the fracture toughness concept and, in practice, it should greatly facilitate finding out its magnitude experimentally.

Also in Fig. 3 can be seen the variation of the critical stress intensity factor when the plate thickness is increased until it provides plain strain conditions. In fact, only from a thickness of $t = 34$ cm onwards does the critical stress intensity factor remains unaltered and can be identified as the fracture toughness of the material.

In this respect, Fig. 4 demonstrates that the lowest thickness necessary for a plain strain performance of the sheet at the crack region varies if the crack size is changed, in the sense that the longer the crack is, the thicker the plate has to be. The figure shows that if the crack is 28 cm long, a thickness of $t = 23$ cm is sufficient for the critical stress intensity factor to become coincident with the fracture toughness whereas, if the crack length is 56 cm, the thickness should be at least $t = 34$ cm. It is also noticeable from Fig. 4 that the fracture toughness estimated for the material by finite element analysis varies if the central crack of the plate has its length changed. In particular, the figure shows that if the half crack goes from 14 cm up to 28 cm within a plate 600 cm wide, the estimated fracture toughness falls from 96263 N/cm$^{3/2}$ down to 95653 N/cm$^{3/2}$. This represents a deviation of only 0.6% with respect to the theory, which predicts that the stress intensity factor and so the fracture toughness remain constant, irrespective of the crack length. In what follows, an intermediate value of the above two, $K_c = 9.6 \times 10^4$ N/cm$^{3/2}$, will be taken as the true one for the purpose of reference.

This result is generalized in Fig. 5 where the fracture toughness, as deduced from testing the largest plate, is plotted versus the crack size. It is true that the finite element analysis advises the use of cracks over a minimum size, say $2a = 20$ cm, to work out the fracture toughness of the plate material. Nevertheless, beyond that, the estimated fracture tougness is effectively almost constant, in accordance with the theory.

As the lowest thickness to guarantee a plain strain performance of the sheet at the crack region is 34 cm, the critical stress intensity factor represented in Fig. 5, corresponding to $t = 35$ cm, is the actual fracture toughness, whereas the critical stress intensity factor, corresponding to $t = 2$ cm, strictly speaking, is not the fracture tougness. Nevertheless, it is remarkable that the deviation in the fracture toughness estimates, originated by using a plate of thickness 2 cm instead of one over 34 cm thick, is only of the order of 1.6%. Consequently, when working out the fracture toughness of any metallic material, we are fully justified in using plates of thickness far less than that required by theory.

Figs. 6 and 7 represent the results obtained with a finite plate 100 cm wide. Basically, they confirm all what has been pointed out from the largest plate results, with respect to the dependence of the critical stress intensity factor and of the fracture toughness upon both the crack length and the thickness of the plate. Besides, as for determining the magnitude of the fracture toughness, attention must be drawn to the important fact that the estimate of the fracture toughness derived from the narrow plate is only 1% over the value deduced from the large plate. This represents a very close agrement between the two measurements and makes clear the validity of using a finite plate, accounting for the appropriate geometry correction factor, when determining the fracture toughness.

Furthermore, it is surprising that even if the narrow sheet is only 2 cm thick, less by far than the required minimum thickness for the plate to behave as plain strain, the critical stress intensity factor can still be assimilated to the fracture toughnes, for the error will be as small as of the order of 3%.

At this point, although this paper focusses on the fracture toughness concept, which corresponds to the critical stress intensity factor under plain strain conditions, something can and must be said with respect to elements behaving as plane stress, such as tensile thin plates. Effectively, Figs. 3, 4 and 6 reveal straightaway that the critical stress intensity factor depends in all cases on the plate thickness, and the thinner the plate gets, the stronger this dependence becomes. This is so to the extent that if, eventually, the plate thickness were diminished to only a few milimeters, which would clearly correspond to plane stress conditions, as the curves of Figs. 3, 4 and 6 become nearly upright, then the critical stress intensity factors would be several times larger than the plane strain fracture resistance. It is remarkable how this fully agrees with what Kanninen and Popelar (1985) stated.

Finally, Fig. 8 shows the minimum thickness required by the three plates for a plain strain performance, depending on the crack size. In fact, after what has been deduced above, the relevance of these graphics is very much reduced in practice. The main consequence of the figure is that for cracks up to 55 cm long, a thickness of $t = 35$ cm should be enough for the plate to behave as plain strain, irrespective of the plate width.

Anyway, the figure is shown for the benefit of a better understanding of the implications of fracture mechanics theory.

7. Conclusions

Finite element methods have proved to be very appropriate to verify the fracture toughness concept, as defined by fracture mechanics.

The dependence of the critical stress intensity factor of a centre cracked plate upon the thickness of the plate and on the crack length has been quantitatively analysed by finite element procedures. This leads to a better understanding of the theoretical concept of fracture toughness and to anticipate under what conditions it can be identified and properly named as the fracture toughness of the plate material.

Furthermore, some drastic conclusions can be drawn to make easier the evaluation of the actual fracture toughness of a material.

Although fracture mechanics theory is strict when defining the plain strain fracture toughness, it has been quantitatively proved that, in practice, the determination of its magnitude can be carried out on a finite plate, accounting for the geometry correction factor. Moreover, the thickness of the plate can be far less than the theoretical requirements, for the subsequent error is very small. This can greatly facilitate the tests for working out the fracture toughness of any alloy and make them cheaper.

References

Atkins, A.G., Mai, Y.W., 1985. Elastic and Plastic Fracture. Wiley, New York.

Broek, D., 1988. The Practical Use of Fracture Mechanics. Kluwer, Dordrecht.

Chen, C.H., Hsu, J., 1997. The stress intensity factors of regularly perturbed-interface cracks of anisotropic bimateirals. Int. J. Solids and Structures 34, 1235–1254.

Dao, W.R., 1996. The stress intensity factors of a rectangular plate with colinear cracks under uniaxial tension. Engng. Fracture Mech. 56, 347–356.

Feddersen, C.E., 1966. Discussion of: Okabe strain crack toughness testing of metallic materials. ASTM STP 410, 77.

Isida, M., 1971. Effect of width and length on stress intensity factors of internally cracked plates under various boundary conditions. Int. J. Fracture Mech. 7, 301–306.

Kanninen, M.F., Popelar, C.H., 1985. Advanced Fracture Mechanics. Oxford University Press, Oxford.

Kim, Y.J., 1996. Stress intensity factors for the annular crack surrounding an elastic fiber under torsion. Int. J. Solids and Structures 34, 2463–2476.

Knott, J.F., 1973. Fundamentals of Fracture Mechanics. Butterworths, London.

Li, Z., 1996. A new technique for determining fracture toughness K_{1c} and its confidence with single DCR specimen. Engng. Fracture Mech. 55, 133–137.

Li, D.M., Bakker, A., 1997. Fracture toughness evaluation using circumferentially cracked cylindrical bar specimens. Engng. Fracture Mech. 57, 1–12.

Liu, C.F., Jou, H.S., Lee, Y.T., 1996. Stress intensity factor of a patched crack. Int. J. Solids and Structures 34, 1557–1562.

Mínguez, J.M., 1993. Study of elastic behaviour of plates containing cracks by finite element analysis. Computers & Structures 47, 917–925.

Oh, H.K., 1996. Determination of fracture toughness by uniaxial tensile test. Engng. Fracture Mech. 55, 865–868.

Parker, A.P., 1981. The Mechanics of Fracture and Fatigue. E. & F.N. Spon Ltd, London.

Seibi, A., Al-Alawi, S.M., 1996. Prediction of fracture toughness using artificial neural networks (ANNs). Engng. Fracture Mech. 56, 311–319.

Sheinman, I., Kardomateas, G.A., 1997. Energy release rate and stress intensity factors for delaminated composite laminates. Int. J. Solids and Structures 34, 451–459.

Sinclair, G.B., Messner, T.W., Meda, G., 1996. Stress intensity factors for deep edge cracks in bending. Engng. Fracture Mech. 55, 19–24.